Mathematical Modelling of Soil Characteristics and Changes Effected by Compaction

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Abstract

Mathematical models have been used for quantitative representation of changes in soil characteristics due to compactive forces. Due to highly complex nature of the problem and difficulties in making accurate observations, only limited success has been reported so far. Very basic, regression type models, are currently available on the relationship between compactive forces and soil bulk density. However, analytical models are available on hydraulic conductivity of soils. Hydraulic conductivity is related to the macropores in soils, which in turn are related to the bulk density. This paper examines the possibility of incorporating the effects of compaction into an analytical model on soil moisture movement.

Introduction

Mathematical models are quantitative representations of physical processes. When an input is fed into a physical system, the components of the system manipulates the input and produces an output. Mathematical models mimic the physical system by a set of equations such that the input values produce the same output values as the physical system when the equations are solved. For example, if we consider a catchment to be a physical system with rainfall as the input and streamflow out of the catchment as the output, a mathematical model of the catchment should be able to manipulate rainfall data with a set of equations and come up with streamflow values which match actual observations within acceptable errors. Since the real world is extremely complex, and our knowledge and our capacity to observe the real world phenomena are limited, mathematical models are essentially simplified versions of actual state of affairs. Some mathematical models seek to find a regression type relationship between input and output ignoring the mechanism which goes on within the system. Such mathematical models are called black box models. An example in the context of catchment modelling is the unit hydrograph. By contrast, white box models or analytical models are attempts to describe the physics of each component of the physical system and find a rationale for the systems behaviour as it does. An analytical catchment model applies the laws of physics to raindrops describing their dispersion under given ambient atmospheric conditions, soil moisture state and infiltration capacity, ground topography, and gravitational forces. The rainwater is traced and its movement described by the laws of physics until the water reaches the outflow point. Considering the variability and heterogeneity of ground surface, soil, and atmospheric conditions some sort of averaging of basin and rainfall characteristics are essential for a realizable situation. SHE model is such an analytical model. The data requirements for an analytical model are enormous, and the costs associated with operation can be colossal. Half-way between the black box and white box models are the grey box or conceptual models. Conceptual models endeavour to describe the physics of the system but introduces fictitious elements to simplify the real world. Tank models are conceptual models for catchments. While the real world does not have a series of tanks containing surface water, subsurface water, and groundwater, but the actual catchment behaves similarly as a series of tanks in terms of output when excited by similar inputs.

Any mathematical model requires data on system characteristics in order to predict output from a given input. For example, to predict runoff from a given rainfall we need information on ground slope, soil porosity, atmospheric humidity, and similar data which are called parameters. Sometimes parameters are measurable and can be measured from the field, but often they cannot be measured because either satisfactory methods of measurements do not exist or any measurement scheme is prohibitively expensive. To find values of parameters which cannot be measured, a procedure called model calibration and verfication is used. The model calibration procedure consists of starting with the plausible values of the parameters and compare model outputs with actual outputs for a given series of inputs; then by a system of trial and error modify the values of the parameters until a good match between observed and model outputs are achieved. To test that the calibrated model continues to perform beyond the input-output data used for calibration, a new set of input-output values are used to assess the performance of the model. The latter process is called model verification. A mathematical model is considered acceptable when its performance at verification stage is similar to that at calibration stage. International standard is that three-fourths of available data be used for calibration and one-fourth for verification. The length of data required for model calibration and verification depends on the complexity of the model, but it should be at least several hundred values.

Mathematical Models for Soil Compaction

Analytical models between degree of soil compaction and compactive forces caused by agricultural machinery do not exist. A conceptual model has been proposed by Arvidsson and Hakansson (1991). They defined ultimate compaction as that achieved by standardized uniaxial compaction of 200 kPa. The ratio of bulk densities before and after ultimate compaction is the degree of compactness. Their model for the degree of compactness (*D*) is:

$$D = 68.2 + 2.37(\text{soil moisture class}) - 7.82(\text{soil moisture class})^2 + 0.002(\text{axle load in kg}) + 4.8\log(\text{inflation pressure in kPa}) - 0.055(\text{tyre width in cm}) \qquad (1)$$

They gave additional formulations for increased number of passes and crop yield losses, but they did not really give a convincing rationale for using the variables in the way they are used in equation (1). McBride (1989) gave the following quasi-theoretical relationship:

$$\ln(\rho) = \ln(\rho_o) - (A + B\sigma)[1 - \exp(C\sigma)] \qquad (2)$$

where ρ is post-compression dry bulk density; ρ_o is pre-compression dry bulk density; σ is total normal stress applied; A,B, and C are parameter coefficients. McBride's formulation does not include soil moisture content, but it has been shown by many investigators (e.g., Voorhees, 1987) that mositure content has a profound influence on the degree of compaction. Larson et al. (1980) gave the following formulation at a given degree of water saturation, S_1, under applied normal stress, σ_a:

$$\rho = [\rho_k + S_T(S_1 - S_k)] + C_\rho \log (\sigma_a/\sigma_k) \qquad (3)$$

Where ρ_k is dry bulk density at known applied stress σ_k and degree of water saturation S_k; S_T is the slope of ρ versus the degree of water saturation at a standard stress of 100 kPa; C_ρ is the compression index.

Compressive forces reduce the macropores in soils. This in turn reduces the hydraulic conductivity of soils affecting the nutrient uptake capability of plants. Fortunately analytical models for hydraulic conductivity of soils have been developed. Combining mass balance equation for incompressible fluid with Darcy's law for flow through porous medium, we get the Richards' equation. The necessary and sufficient conditions for use of this equation in non-swelling soils are that both the hydraulic conductivity, K, and water potential, ψ, are well-defined macroscopic average functions of the volumetric water content θ. Richards' equation is nonlinear partial differential equation and cannot be solved directly except for very simple boundary conditions. The available computer software SWIM solves the Richards' equation by numerical methods, and in the following section we describe the SWIM model and how we can incorporate the effects of compaction in the model.

The SWIM model

The SWIM model solves the following partial differential equation by a fixed grid implicit numerical scheme (Ross, 1990):

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} K\left(\frac{\partial \psi}{\partial x} + \frac{\partial z}{\partial x}\right) + s \qquad (4)$$

where θ is volumetric water content, t is time, s is source strength, ψ is matric potential, z is gravitational potential, and x is the direction of water movement or hydraulic gradient. For computational efficiency SWIM assumes horizontal homogeneity, negligible vapour flow in soil, no temperature changes, and no hysteresis in soil moisture curves. The model uses a very simple power law for overland flow, and simply the product of potential evaporation and surface matric potential to estimate evapotranspiration. Evidently, the model is very crude in terms of overland flow and evapotranspiration, but those are not the important components of the model and major errors are not expected when those elements are not significant. The important component of the model is infiltration and deep drainage. The input requirements of the model are the following variables:

i) simulation time
ii) vegetation characteristics (xylem potential, root development, transpiration)
iii) soil moisture characteristics (matric potential, air entry potential, moisture content)
iv) unsaturated hydraulic conductivity function
v) surface conductance and rainfall distribution
vi) runoff coefficients
vii) cumulative rainfall and
viii) cumulative evaporation

The printed outputs from the model contain the following variables:

i) potential evaporation (eps) for the soil
ii) potential transpiration (tp) for the vegetation
iii) actual evaporation (eas) for the soil
iv) actual transpiration (ta) for the vegetation
v) water potentials (psi) for the soil depths
vi) cumulative rainfall (cr)
vii) cumulative runoff (cro)
viii) cumulative drainage (cd)
ix) cumulative evapotranspiration [ce(cev + cevs)]
x) surface water (surf)
xi) total water present in the soil profiles (pres)

Incorporation of Compaction Effect into SWIM model:

In hydraulic properties function, the following equation is used to calculate the water content

$$\theta/\theta_s = (\varphi/\varphi_e)^{-1/b} \qquad (5)$$

where φ_e is the air entry water potential (potential at which the largest water filled pores just drain), and b is the slope. In order to incorporate the effect of tillage or compaction on hydraulic properties, and to take account of density effects on moisture retention, the effect of bulk density (ρ_b) can be specified on the parameters in equation (5). Hall et al (1977, cited by Campbell, 1985) gave an empirical relation as

$$\varphi_e = \varphi_{es}(\rho_b/1.3)^{0.67b} \qquad (6)$$

where φ_{es} is the air entry potential and it is for a standard bulk density of 1.3 Mg/m^3 and can be calculated from the equation

$$\varphi_{es} = -0.5 d_g^{-1/2} \qquad (7)$$

and b can be calculated from the relation given below

$$\mathrm{b} = -2\,\varphi_{es} + 0.2\,\sigma_g \qquad (8)$$

where d_g in equation (7) is the geometrical particle diameter and σ_g in equation (8) is the geometrical standard deviation both of those can be calculated from

$$d_g = \exp(\alpha)$$

$$\sigma_g = \exp(\beta)$$

where $$\alpha = \Sigma m_i \ln(d_i)$$

$$\beta = \sum m_i \ell n^2(d_i) - \alpha^2]^{1/2}$$

m_i is the mass fraction of textural class i and d_i is the arithmetic mean diameter of class i.

Results and Discussion

Ongoing field studies are expected to provide adequate data to test the significance of compaction on hydraulic characteristics of soil mass, and the results will be reported elsewhere. To give a feel as to how hydraulic characteristics may change due to application of compactive forces on soil mass, the original and modified SWIM models were fed with different bulk density and saturated hydraulic conductivity values as observed in the field and the results are summarised in Table 1. All the values show significant effect due to model change. The greatest effect being in case of low bulk densities. This difference may be due to the fact that the original model uses air entry potential value from soil moisture retention curve, whereas the modified model calculates the same from bulk density. Since the soil moisture retention curve used is hypothetical, a conclusive statement regarding model performances cannot be made.

References

Arvidsson, J. and Hakansson, I. (1991), "A model for estimating crop yield losses caused by soil compaction." *Soil & Tillage Research*. Vol. 20, pp. 319-332.

Campbell, G.S. (1985), *Soil Physics with Basic. Transport Models for Soil-Plant Systems*. Elsevier, New York

Larson, W.E., Gupta, S.C., and Useche, R.A. (1980), "Compression of agricultural soils from eight soil orders." *Soil Science Society of America Journal*. Vol. 44, pp. 450-457.

McBride, R.A. (1989), "Estimation of density-moisture-stress functions from uniaxial compression of unsaturated, structural soils." *Soil & Tillage Research*. Vol. 13, pp. 383-397.

Ross, P.J. (1990), "SWIM - A simulation model for soil water infiltration and movement." Reference Manual. CSIRO Division of Soils, Townsville, Queensland.

Voorhees, W.B. (1987), "Assessment of soil susceptibility to compaction using soil and climatic data bases." *Soil & Tillage Research*. Vol. 10, pp. 29-38.

Table 1. Comparsion of SWIM and modified SWIM outputs

Model used	Bulk density, gm/cc		K_s,cm/h		Rainfall cm	Runoff cm	Infiltration cm	Drainage cm	ET cm	Profile water, cm
	Surface	15 cm depth	Surface	15cm Depth						
SWIM	1.09	1.16	48.93	15.52	30.00	0.00	30.00	7.61	22.44	1.75
Modified	1.09	1.16	48.93	15.52	30.00	0.00	30.00	7.34	22.65	1.37
SWIM	1.38	1.35	1.57	0.88	30.00	0.00	30.00	6.73	23.40	1.67
Modified	1.38	1.35	1.57	0.88	30.00	0.00	30.00	6.66	23.66	1.66
SWIM	1.40	1.36	1.17	0.78	30.00	0.00	30.00	6.68	23.46	1.66
Modified	1.40	1.36	1.17	0.78	30.00	0.00	30.00	6.62	23.52	1.67